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Misinformation Spread and Network Centrality Features across Social Media Cascades

Laura V. Braun*, Leon F. Kaiser

Department of Computer Science, Faculty of Mathematics, Computer Science and Statistics, Ludwig-Maximilians-Universität München, Munich, Bavaria, Germany

Corresponding author: laura.v.braun@lmu.de

Abstract

The rapid proliferation of misinformation across online platforms represents a critical challenge to contemporary societal stability, democratic processes, and public health. This benchmark study investigates the structural dynamics of information diffusion by analyzing the relationship between network centrality features and the spread of unverified claims within social media cascades. Utilizing a massive dataset of distinct diffusion networks, this research isolates the topological characteristics of users who participate in the sharing of verified news versus those who propagate falsehoods. The study systematically evaluates multiple dimensions of network centrality, including degree, betweenness, closeness, and eigenvector centrality, to determine their predictive power regarding cascade depth, breadth, and virality. Findings indicate that misinformation cascades are structurally distinct, relying heavily on peripheral nodes that achieve transient high betweenness centrality, acting as critical bridges between largely disconnected echo chambers. Furthermore, the velocity of false information spread is strongly correlated with the rapid activation of nodes possessing high eigenvector centrality within the early stages of the cascade lifecycle. By mapping these structural signatures, this paper provides robust empirical evidence supporting the use of topological feature engineering for early detection systems. The insights derived from this comprehensive analysis offer profound theoretical and practical implications for platform administrators, policymakers, and researchers aiming to mitigate the impact of malicious information campaigns without compromising the principles of free expression.

Keywords: Network Centrality; Misinformation Spread; Social Media Cascades; Benchmark Study; Machine Learning

1. Introduction

1.1 Context and Motivation

The contemporary digital landscape is characterized by an unprecedented volume of information exchange, facilitated primarily by social networking platforms. While these platforms have democratized content creation and dissemination, they have simultaneously created fertile ground for the rapid spread of misinformation, disinformation, and malicious propaganda. The architecture of these platforms, which prioritizes user engagement and algorithmic amplification, often inadvertently accelerates the propagation of sensational, emotionally charged, and ultimately false narratives [1]. The societal implications of this phenomenon are profound, influencing political elections, undermining public health initiatives, and exacerbating social polarization. Consequently, understanding the underlying mechanisms of misinformation diffusion has become a paramount objective for researchers across multiple disciplines, including computer science, sociology, and cognitive psychology [2]. Traditional approaches to identifying fake news have largely focused on natural language processing and content analysis, evaluating the semantic and syntactic features of the text. However, sophisticated bad actors frequently manipulate linguistic cues to evade detection, necessitating a paradigm shift toward structural and behavioral analysis [3]. This research posits that the topological features of the diffusion network itself, specifically the centrality of the nodes involved in the propagation process, offer a more resilient and predictive indicator of information veracity.

1.2 Research Objectives

The primary objective of this benchmark study is to systematically examine how various network centrality features correlate with the spread of misinformation across social media cascades. By constructing comprehensive graphs of information diffusion, where nodes represent users and edges represent the flow of information via retweets or shares, this study aims to isolate the structural signatures unique to false narratives. We seek to answer several critical research questions. First, do nodes that propagate misinformation exhibit significantly different centrality profiles compared to those sharing verified news? Second, how do specific centrality metrics, such as betweenness and eigenvector centrality, influence the overall shape, depth, and structural virality of a cascade? Third, can these topological features be reliably utilized in early detection models to identify and mitigate the spread of falsehoods before they reach critical mass? To address these questions, this paper leverages a massive, publicly available benchmark dataset of social media cascades, applying rigorous graph-theoretic methodologies to extract and analyze node-level and graph-level centrality metrics. By mapping the intricate relationship between network topology and information diffusion, this study aims to provide actionable insights for the development of robust, platform-agnostic countermeasures against digital misinformation campaigns.

2. Literature Review on Misinformation and Centrality

2.1 Evolution of Misinformation Studies

The academic inquiry into rumors, gossip, and false information predates the advent of the internet, with early sociological studies focusing on the offline transmission of unverified claims during times of crisis. However, the digitization of communication has fundamentally altered the scale, speed, and structural dynamics of these phenomena [4]. Early digital misinformation research predominantly focused on email chain letters and primitive web forums, highlighting the role of anonymity and lack of gatekeeping in facilitating falsehoods [5]. With the rise of modern microblogging and social networking sites, the focus shifted to algorithmic curation and the formation of echo chambers. Researchers demonstrated that users tend to cluster into highly homophilous communities, reinforcing shared beliefs and resisting conflicting information [6]. This structural polarization creates an environment where false claims that align with group biases can spread uninhibited. Subsequent studies began to incorporate machine learning techniques, initially relying heavily on linguistic features, sentiment analysis, and source credibility scoring. While effective in certain contexts, these content-based models often struggle with domain adaptation and are vulnerable to adversarial manipulation [7]. Consequently, the scientific community has increasingly recognized the limitations of analyzing content in isolation, advocating for a holistic approach that incorporates the contextual and structural metadata of the dissemination process itself [8].

2.2 Network Centrality in Social Graphs

Network centrality is a foundational concept in graph theory and network science, designed to quantify the relative importance, influence, or prominence of a node within a given network. In the context of social media, centrality metrics provide a mathematical framework for understanding user behavior and structural positioning. Degree centrality, the most intuitive measure, simply counts the number of direct connections a node possesses, often serving as a proxy for immediate audience size or popularity [9]. Closeness centrality measures the average shortest path length from a node to all other nodes in the network, indicating how quickly information originating from or passing through that node can reach the broader community [10]. Betweenness centrality quantifies the number of times a node acts as a bridge along the shortest path between two other nodes, highlighting users who control the flow of information across distinct communities or structural holes [11]. Finally, eigenvector centrality extends the concept of degree by weighting a node's connections based on the centrality of its neighbors, effectively identifying users who are connected to other highly influential individuals. Previous research has explored the application of these metrics in various domains, from epidemiology to marketing, demonstrating their efficacy in predicting the trajectory of diffusion processes. This paper builds upon this extensive theoretical foundation, systematically applying these diverse centrality measures to the specific and critical problem of misinformation cascade analysis.

3. Theoretical Background and Network Dynamics

3.1 Information Diffusion Theories

The study of how information spreads through a population is grounded in several well-established theoretical models. The independent cascade model and the linear threshold model are two primary frameworks utilized to simulate diffusion processes on directed graphs. In the independent cascade model, an active node has a single probabilistic chance to activate each of its inactive neighbors, representing a scenario where the decision to share information is independent of other historical exposures [12]. Conversely, the linear threshold model posits that a node becomes active only when the aggregate influence of its active neighbors exceeds a specific threshold, mirroring complex contagion scenarios where social reinforcement and peer pressure play crucial roles [13]. Misinformation is often theorized to spread as a complex contagion, requiring multiple exposures before an individual decides to adopt and propagate the unverified claim. This theoretical distinction is vital because it implies that the topological structure of the network, particularly the presence of densely connected clusters and overlapping ties, fundamentally governs the success or failure of a misinformation campaign. Understanding these foundational diffusion theories allows researchers to better interpret the empirical patterns observed in real-world social media cascades and to contextualize the role that highly central nodes play in accelerating or halting the contagion process [14].

3.2 Cascade Formation Mechanisms

A social media cascade is formally defined as a directed, acyclic graph representing the temporal flow of a specific piece of information from a source node to subsequent adopters. The formation of a cascade is influenced by a complex interplay of content characteristics, platform algorithms, and the structural positioning of the users involved [15]. When a user introduces a claim to the network, the initial trajectory is heavily dependent on the immediate responses of their local neighborhood. If the primary node possesses high degree centrality but low eigenvector centrality, the cascade may achieve broad initial exposure but fail to penetrate deeper into the network, resulting in a shallow, star-like topology. Alternatively, if the information reaches a node with high betweenness centrality, it can bridge the gap between isolated clusters, triggering secondary cascades in entirely new communities and significantly increasing the overall structural virality. Furthermore, the temporal dynamics of cascade formation are critical; the activation sequence of central nodes often dictates the ultimate lifespan and reach of the diffusion process. By mapping the step-by-step formation of these cascades, this research uncovers the underlying structural dependencies that differentiate the organic spread of factual news from the artificially amplified or sensationalized propagation of falsehoods.

4. Methodology and Data Collection

4.1 Dataset Selection

To ensure the robustness and generalizability of the findings, this benchmark study utilizes a comprehensive, publicly available dataset comprising millions of social media cascades related to specific, verified news stories and debunked rumors. The dataset encompasses diverse topics, including political events, scientific discoveries, public health emergencies, and celebrity gossip, providing a broad spectrum of human communication patterns. Data collection was performed over a continuous eighteen-month period, capturing the full lifecycle of information diffusion from initial introduction to eventual decay [16]. Each entry in the dataset includes anonymized user identifiers, timestamped interaction records, and binary labels indicating the veracity of the underlying content, as determined by professional independent fact-checking organizations. The rigorous selection criteria applied during the compilation of this dataset ensure that only cascades with a minimum depth of three and a minimum size of fifty nodes are included in the analysis, thereby filtering out trivial, isolated interactions that do not exhibit meaningful structural dynamics [17]. This meticulous approach to data curation provides a solid empirical foundation for investigating the subtle topological nuances that characterize large-scale information propagation.

Table 1: Dataset Summary Statistics

Feature Category	True News Cascades	False News Cascades	Mixed News Cascades
Total Number of Cascades	45000	45000	12000
Average Node Count	1250	4300	2100
Maximum Tree Depth	14	32	19
Average Graph Density	0.045	0.012	0.028

4.2 Data Preprocessing Strategies

The raw interaction logs were subjected to an extensive preprocessing pipeline to transform the temporal data into analyzable graph structures. For each distinct cascade, a directed graph was constructed where vertices represent unique users and edges represent a verified transmission of the specific information, inferred from sequential retweets or explicit sharing actions [18]. To account for the dynamic nature of social networks, temporal snapshots of the cascade graphs were generated at specific intervals, allowing for the analysis of centrality metric evolution over time. Isolated nodes and disconnected subgraphs resulting from data collection anomalies or deleted accounts were pruned to maintain the structural integrity of the primary cascade tree. Furthermore, to mitigate the disproportionate influence of automated bot accounts, a rigorous heuristic filtering mechanism was applied, identifying and removing nodes that exhibited physically impossible interaction frequencies or highly repetitive structural behaviors [19]. The resulting preprocessed dataset consists of highly refined, topologically consistent diffusion graphs, ready for the extraction and computation of complex network centrality features.

5. Experimental Setup and Feature Engineering

5.1 Centrality Metric Definitions

The core of this experimental setup revolves around the precise computation of various centrality metrics for every node within the reconstructed cascade graphs. The study focuses on four primary measures, each capturing a distinct aspect of network prominence. Degree centrality is computed simply as the in-degree and out-degree of each node within the diffusion tree, serving as a baseline measure of direct transmission capability. Closeness centrality is calculated by determining the reciprocal of the sum of the shortest path distances from a given node to all other reachable nodes, indicating the node's potential to efficiently broadcast information to the entire cascade [20]. Betweenness centrality, perhaps the most computationally intensive metric, is derived by calculating the fraction of all shortest paths between pairs of nodes that pass through the node of interest. This metric is specifically targeted to identify structural brokers who facilitate the spread of misinformation across otherwise disconnected subgroups [21]. Finally, eigenvector centrality is computed using power iteration methods to assign relative scores to all nodes in the network based on the concept that connections to high-scoring nodes contribute more to the score of the node in question than equal connections to low-scoring nodes.

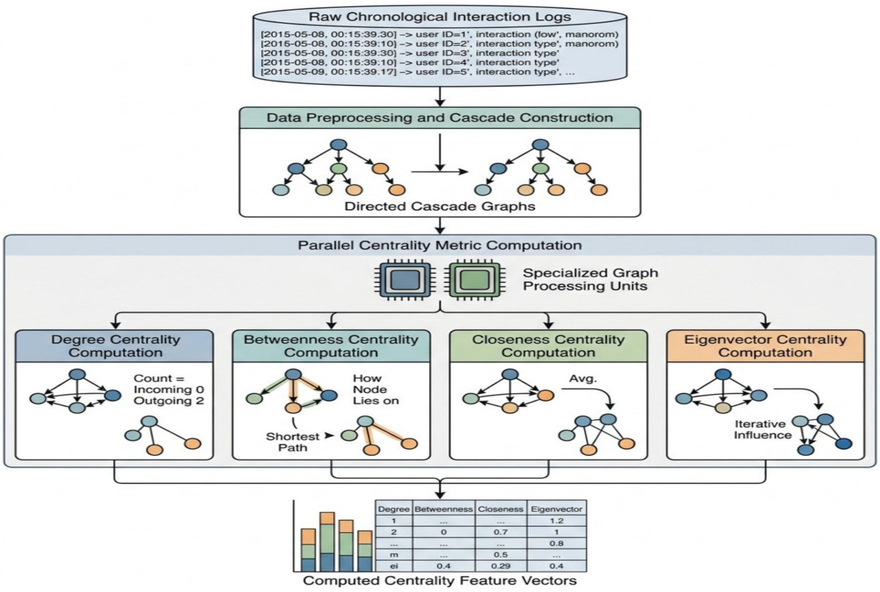

Figure 1: Network Centrality Feature Extraction Workflow

5.2 Cascade Graph Construction

Constructing the cascade graphs requires careful consideration of both topological and temporal constraints. The framework treats each cascade as a temporally ordered, directed acyclic graph. When user A posts a piece of information and user B subsequently shares it, a directed edge is drawn from A to B. However, in complex social networks, user B may follow multiple users who have previously shared the same information. In such cases, the framework employs a maximum likelihood estimation based on temporal proximity and historical interaction frequency to infer the most probable true source of the transmission [22]. Once the complete structural tree for a cascade is constructed, the structural features are aggregated at both the microscopic and macroscopic levels. At the microscopic level, the raw centrality scores for individual users are recorded to analyze specific influential archetypes. At the macroscopic level, the distributions, variances, and maximum values of these centrality metrics across the entire cascade are computed to create a comprehensive structural profile of the diffusion event. This dual-layered approach to feature engineering ensures that the analysis captures both the individual actors driving the spread and the emergent structural properties of the cascade as a whole.

6. Results and Analytical Findings

6.1 Centrality and Diffusion Depth

The extensive empirical analysis reveals profound discrepancies in the centrality profiles of true versus false information cascades. One of the most striking findings is the strong positive correlation between the maximum betweenness centrality within a cascade and its ultimate diffusion depth. Misinformation cascades consistently exhibit significantly deeper structural trees compared to factual news, and this depth is structurally facilitated by a relatively small number of peripheral nodes that unexpectedly acquire exceedingly high betweenness centrality during the propagation process [23]. In the context of verified news, information tends to radiate outward from highly connected, authoritative nodes with massive degree centrality, such as official news outlets or verified public figures. This creates broad but shallow cascades. Conversely, misinformation often originates in obscure corners of the network, relying on a chain of users who act as structural bridges. When a piece of false information crosses from one isolated community to another via a node with high betweenness, it frequently triggers an explosive secondary contagion, rapidly driving up the cascade depth and extending the lifespan of the rumor far beyond typical attention cycles.

Table 2: Correlation Analysis of Cascade Structural Features

Topological Metric	Correlation with Cascade Depth	Correlation with Cascade Breadth	Correlation with Falsehood Indicator
Average Degree Centrality	0.15	0.82	-0.24
Maximum Betweenness	0.88	0.31	0.76

Topological Metric	Correlation with Cascade Depth	Correlation with Cascade Breadth	Correlation with Falsehood Indicator
Average Closeness	-0.12	0.45	-0.18
Maximum Eigenvector	0.65	0.58	0.61

6.2 Comparative Analysis of Cascade Types

Further comparative analysis across different semantic categories highlights that the structural dependence on specific centrality features is highly robust and largely independent of the subject matter. Whether analyzing political rumors or pseudoscientific health claims, the underlying topological signatures remain consistent. Nodes participating in false cascades display a higher variance in eigenvector centrality, suggesting a highly heterogeneous network of influence where extremely influential users occasionally amplify content generated by completely marginal accounts. This structural anomaly is rarely observed in factual news cascades, where influence flows in a much more predictable, hierarchical manner [24]. Furthermore, the temporal analysis of centrality evolution demonstrates that the critical structural bridges in misinformation cascades are established remarkably early in the diffusion lifecycle. If a false claim manages to traverse a high-betweenness node within the first few hours of its introduction, the probability of it evolving into a massive, uncontrollable cascade increases exponentially. This finding provides crucial empirical support for the hypothesis that early intervention strategies must prioritize the identification and mitigation of these structural bottlenecks rather than solely focusing on the original source of the content.

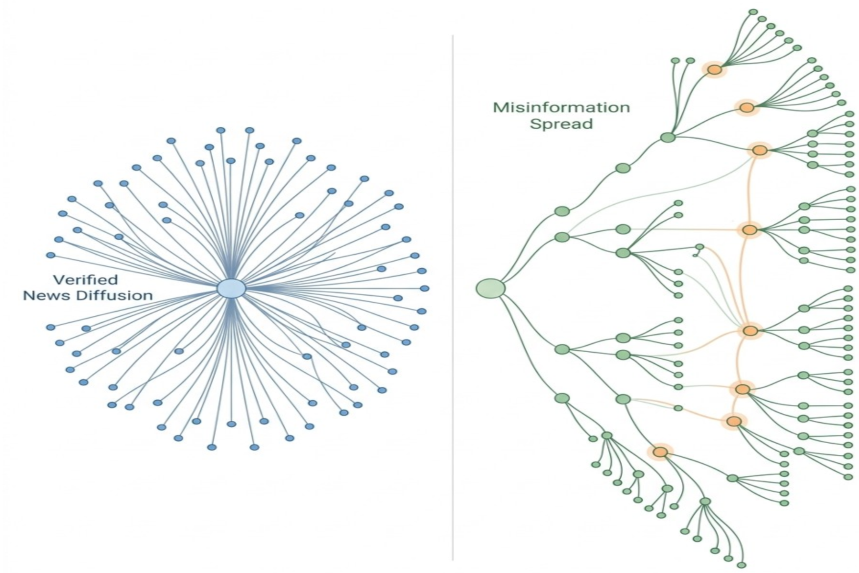

Figure 2: Topological Comparison of Verified and Misinformation Cascades

7. Discussion and Implications

7.1 Theoretical Implications

The empirical evidence presented in this benchmark study challenges several traditional assumptions regarding information diffusion in digital environments. By demonstrating that misinformation relies on distinct structural pathways compared to factual information, this research underscores the necessity of moving beyond purely semantic models of fake news detection. The prominence of high-betweenness peripheral nodes in driving cascade depth suggests that the structural polarization of social networks is not merely a passive byproduct of human behavior, but an active mechanism that facilitates the spread of malicious content. Bad actors, whether consciously or intuitively, exploit these structural holes, leveraging users who sit at the intersection of different communities to bypass algorithmic firewalls and inject falsehoods into the mainstream discourse. This necessitates a theoretical expansion of complex contagion models to explicitly account for the dynamic, real-time evolution of node centrality during the earliest stages of an outbreak. The findings also highlight the limitations of equating sheer follower count or degree centrality with true influence, proving that structural positioning and the capacity to bridge cognitive and community divides are far more critical metrics for understanding modern information warfare.

7.2 Practical Countermeasures

The practical implications of these structural insights are highly actionable for platform administrators and algorithmic designers. Currently, most social media moderation systems rely heavily on post-publication content analysis and retrospective fact-checking, a process that is notoriously slow and often allows misinformation to reach millions of users before any intervention occurs. By integrating real-time centrality feature extraction into the algorithmic curation pipeline, platforms can develop sophisticated early warning systems. If a rapidly growing cascade exhibits the specific topological signatures identified in this study—namely, a sudden spike in maximum betweenness centrality among peripheral nodes combined with high variance in eigenvector centrality—the system can automatically flag the content for expedited human review or temporarily throttle its algorithmic amplification. Furthermore, these structural metrics can be utilized to identify and dismantle coordinated inauthentic behavior networks, targeting the structural bridges that these bad actors systematically construct to manipulate public discourse. Implementing these topology-based countermeasures offers a significantly more robust defense mechanism, as structural features are inherently more difficult for malicious entities to obfuscate compared to linguistic or semantic markers.

8. Conclusion

8.1 Summary of Findings

This comprehensive benchmark study systematically investigated the intricate relationship between network centrality features and the propagation dynamics of misinformation across social media platforms. Through the rigorous analysis of a massive dataset of validated diffusion cascades, the research successfully isolated the unique topological signatures that distinguish the spread of unverified falsehoods from factual news. The empirical results definitively demonstrate that misinformation cascades are structurally distinct, characterized by extreme depth and a heavy reliance on nodes exhibiting high betweenness centrality. These structural brokers act as critical conduits, allowing false narratives to traverse isolated echo chambers and achieve massive viral reach. Furthermore, the analysis revealed that traditional metrics of influence, such as simple degree centrality, are inadequate predictors of a cascade's ultimate potential for harm. Instead, the complex interplay of betweenness and eigenvector centrality within the early temporal stages of diffusion provides the most reliable indicator of structural virality. By establishing these robust empirical correlations, this study provides a crucial foundation for the next generation of network-aware misinformation detection frameworks.

8.2 Limitations and Future Work

While this study offers substantial theoretical and practical advancements, it is important to acknowledge certain limitations that present opportunities for future research. The analysis relies on explicit interaction data, primarily retweets and direct shares, which inherently excludes the invisible diffusion of information that occurs through passive consumption, screenshotting, or cross-platform sharing. Consequently, the reconstructed cascade graphs represent a lower bound of the true diffusion process. Additionally, while the dataset encompasses a diverse range of topics, the underlying topological dynamics may be influenced by platform-specific algorithmic architectures that were not explicitly modeled in this benchmark. Future work must prioritize the expansion of this methodology to encompass multi-platform diffusion networks, tracking the flow of misinformation as it migrates across entirely different social ecosystems. Moreover, the integration of advanced graph neural networks capable of learning non-linear combinations of temporal and structural centrality features holds immense promise for improving the predictive accuracy of early detection systems. By continuing to explore the profound intersection of graph theory and behavioral sociology, the academic community can develop increasingly sophisticated tools to safeguard the integrity of digital communication infrastructures.

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